EK381 Summer 2025 Day 3 Test

Name: _____ BU ID: _____

Question One (Counting)

You have a bag of 12 jelly beans: 6 of them are red, 4 are yellow, and 2 are green.

You select 3 beans from the bag, without looking and without replacing them. What is the probability that they are all red?

Each selection is equally likely.

Solution:

$\mathbb{P}[\text{all 3 are red}] = \frac{\binom{6}{3}}{\binom{12}{3}} = \frac{20}{220} = 1/11$

Question Two (True/False)

Indicate whether the statement is always true or it can be false by clearly writing "True" or "False". You are welcome to briefly explain your reasoning for partial credit (in case your choice is wrong).

If A and B are independent, $\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B]\,\mathbb{P}[A^c]$.

Solution:

True

$P[A \cup B] = P[A] + P[B] - P[A \cap B]P[A] + P[B] - P[A]P[B] = P[A] + P[B](1 - P[A]) = P[A] + P[B]P[A^c]$

Question Three (Independence)

Consider independent events A, B, and C satisfying $\mathbb{P}[A] = 1/2$, $\mathbb{P}[B] = 1/4$, $\mathbb{P}[C] = 2/3$.

Calculate $\mathbb{P}[A \cap C^c | B]$

Solution:

$\mathbb{P}[A \cap C^c | B] = \mathbb{P}[A \cap C^c] = \mathbb{P}[A] \cdot \mathbb{P}[C^c] = (1/2)(1/3) = 1/6$

Calculate $\mathbb{P}[A \cup B \cup C]$

Solution:

$\mathbb{P}[A \cup B \cup C] = 1 - \mathbb{P}[A^c \cap B^c \cap C^c] = 1 - (1/2)(3/4)(1/3) = 7/8$