EK381 Summer 2025 Day 2 Test

SOLUTIONS

Question One

For each of the following parts, indicate whether the statement is always true or it can be false by clearly writing "True" or "False". You are welcome to briefly explain your reasoning for partial credit (in case your choice is wrong).

If $\mathbb{P}[A] + \mathbb{P}[B] + \mathbb{P}[C] = \mathbb{P}[\Omega]$, then A, B, and C are a partition.

Solution:

False. Setting $A = B = C$ and $\mathbb{P}[A] = 1/3$ shows that $A \cup B \cup C$ is not a partition.

If A, B, and C are a partition, then

$\mathbb{P}[A^c] + \mathbb{P}[B^c] + \mathbb{P}[C^c] = 2$

Solution:

True.

$\mathbb{P}[A^c] + \mathbb{P}[B^c] + \mathbb{P}[C^c] = 1 - \mathbb{P}[A] + 1 - \mathbb{P}[B^c] + 1 - \mathbb{P}[C] = 3 - (\mathbb{P}[A] + \mathbb{P}[B] + \mathbb{P}[C]) = 2.$

Question Two

Let A and B be events with $\mathbb{P}[A|B] = 1/2$, $\mathbb{P}[A|B^c] = 1/4$ and $\mathbb{P}[B] = 1/4$.

Calculate $\mathbb{P}[B \cap A]$.

Solution:

$\mathbb{P}[B \cap A] = \mathbb{P}[A|B]\mathbb{P}[B] = (1/2)(1/4) = 1/8$

Calculate $\mathbb{P}[B|A]$

Solution:

$\mathbb{P}[B|A] = \frac{\mathbb{P}[B \cap A]}{\mathbb{P}[A]}$

$= \frac{1/8}{\mathbb{P}[A|B]\mathbb{P}[B] + \mathbb{P}[A|B^c]\mathbb{P}[B^c]} = \frac{1/8}{1/8 + (1/4)(3/4)} = \frac{2/16}{2/16 + 3/16} = 2/5$