Homework 12: finish by 6/25

Reading: Notes: Chapter 11

Videos: 11.1 - 11.4

Problem 12.1 (Video 11.1 - 11.4, Lecture Problem)



- (a) Write down the state transition matrix **P**.
- (b) Determine the 2-step state transition matrix $\mathbf{P}(2)$. You can use calculators or MATLAB for this computation.
- (c) What are the state probability vectors \underline{p}_1 and \underline{p}_2 ?
- (d) Evaluate $\mathbb{P}[X_0 = 1, X_1 = 2, X_2 = 2]$ and $\mathbb{P}[X_0 = 3, X_1 = 1, X_2 = 2]$.

Problem 12.2 (Video 11.1 - 11.4)

Consider a Markov chain with the following state transition matrix and initial probability state vector:

	0	1	0		[1/3]	
$\mathbf{P} =$	0	1/3	2/3	$p_0 =$	1/3	
	1	0	0	_0	$\lfloor 1/3 \rfloor$	

- (a) Draw the Markov chain, labeling the states as 1, 2, and 3, as well as labeling the arcs with the appropriate transition probabilities.
- (b) What is the period of state 1?
- (c) Determine $\mathbb{P}[X_0 = 2, X_1 = 2, X_2 = 3].$
- (d) Does a unique limiting state probability vector $\underline{\pi}$ exist? If so, argue why and solve for it. If not, argue why.

Problem 12.3 (Video 11.1 - 11.4, Lecture Problem)

Consider the following discrete-time Markov chain with initial state 3.



- (a) What are the communicating classes?
- (b) For each communicating class, determine the period and whether it is transient or recurrent.
- (c) Write down the state transition matrix **P**.
- (d) Does a unique limiting state probability vector $\underline{\pi}$ exist? If so, argue why and solve for it. If not, argue why.

Problem 12.4 (Video 11.3) Consider the following Markov chain. For each of the parts below, you only need to know that each arc represents a positive probability.



- (a) Determine the communicating classes.
- (b) Determine the period for each communicating classes.
- (c) Determine which communicating classes are recurrent and which ones are transient.

Problem 12.5 (Video 11.1 - 11.4)

Consider a 4 state Markov chain with the transition probability matrix

$$\mathbf{P} = \begin{pmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0 & 0.5 & 0.2 & 0.3 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.1 & 0.9 \end{pmatrix}$$

- (a) Draw the state transition diagram, with the probabilities for the transitions.
- (b) Find the transient states and recurrent states.
- (c) Is the Markov chain irreducible? Explain.
- (d) Is the Markov chain aperiodic? Explain.
- (e) Find the steady state distribution of this Markov chain.