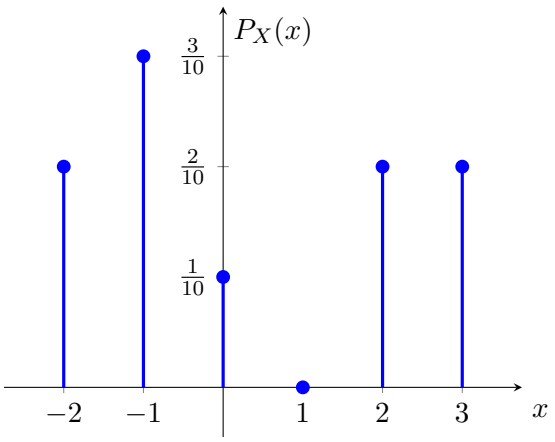
Homework 3: finish by 5/30.

Reading: Notes: Chapter 2.

Videos: 2.3 - 2.6

Problem 3.1 (**Video 2.3 - 2.6**, **Lecture Problem**)



Let X be a discrete random variable with probability mass function (PMF) as above. Let event $A = \{-2, 1, 3\}$.

(a) Given that $\{X \in A\}$ occurs, what is the conditional probability that $X > 1$, that is $\mathbb{P}[X > 1 | X \in A]$?

(b) Determine $\mathbb{E}[X]$ and $\mathbb{E}[3X + 2]$.

(c) Determine $\mathsf{Var}[X]$ and $\mathsf{Var}[2X - 1]$.

Problem 3.2 (**Video 2.3 - 2.6**, **Quick Calculations**)

Calculate each of the requested quantities.

(a) Your favorite sports team wins a game with probability $\frac{3}{5}$, independently of other games. Let X be the number of games they win out of 20. What kind of random variable is X? (Don't forget the parameters.) Calculate $\mathbb{E}[X]$ and $\mathbb{E}[X^2]$.

(b) Let X be Poisson(λ) and assume that $\mathbb{E}[X] = 2$.
Calculate λ, and $\mathbb{P}[X \leq 3]$ and $\mathbb{P}[X \leq 3 | X > 0]$.

(c) Roll a six-sided die until the first 2 appears. Let X denote the number of rolls. What kind of a random variable is X? (Don't forget the parameters.) Calculate $\mathbb{E}[2X - 1]$ and $\mathsf{Var}[2X - 1]$.

(d) Let X be a random variable with $\mathbb{E}[X] = -1$ and $\mathsf{Var}[X] = 4$. Let $Y = -2X + 3$. Calculate $\mathbb{E}[Y]$ and $\mathsf{Var}[Y]$.

(e) Let X be a random variable with $\mathbb{E}[X] = 0$ and $\mathsf{Var}[X] = 2$. Calculate $\mathbb{E}[X^2]$ and $\mathbb{E}[(2X-1)^2]$.

Problem 3.3 (**Video 2.5, 2.6**, **Lecture Problem**, **Spring 2022 Exam 1 Problem**)

You are measuring the number of spikes from a neuron in a one-second window. The resulting random variable X is Poisson(λ).

(a) After careful study, you have determined that the average number of spikes observed from this neuron in one second is $\mathbb{E}[X] = 2$. What is the probability that you see no spikes at all in a one-second window?

(b) What is the probability that the number of spikes you see in a one-second window is *less than or equal* to average? (Recall from (a) that the average is 2.)

(c) Calculate $\mathbb{E}[3X^2 + 2X + 1]$.

(d) Given that the number of spikes in a one-second window is *less than or equal* to average, what is the conditional expected value of X?

Problem 3.4 (**Lecture Problem**, **Video 2.5, 2.6**) You have started watching Game of Thrones (or House of the Dragon), and from the very beginning realize that a particular character is your favorite. However, you are aware of the show's reputation for killing off characters, and would like to calculate the probability your favorite is eliminated after a certain number of episodes. Specifically, you use the following model: for each episode, the probability that your character is eliminated is 1/3, independently of all other episodes. Let X be the episode number where your character is eliminated.

(a) What kind of random variable is X? (Don't forget the parameters.)

(b) What is the probability that your character is eliminated in the third episode?

(c) What is the probability that your character lasts at least two episodes?

(d) Let B be the event that your character lasts at least two episodes. Determine the conditional PMF of X given event B.

(e) Given that your character lasts at least two episodes, what is the probability that they are eliminated in the third episode?

(f) Given that your character lasts at least two episodes, what is the probability that they last four or more episodes?

Problem 3.5 (**Video 2.5, 2.6**, **Fall 2020 Exam 1 Problem**)

You are practicing your free throws for an upcoming basketball game. Every throw is successful with probability 2/3, independently of the others. Let X denote the number of successful throws out of 5.

(a) What kind of random variable is X? (Don't forget the parameters.)

(b) What is the probability that you successfully make at least 3 out of the 5 free throws?

(c) Given that you successfully make at least 3 out of 5 free throws, what is the probability that you successfully make exactly 3 out of 5?

(d) What is the probability of scoring exactly 3 **consecutive** free throws within the set of 5?

(e) You keep practicing with sets of 5 free throws. What is the average number of sets until your first set where you miss every free throw?