EK381 Probability, Statistics, and Data Science for Engineers | Boston University
Prof Carruthers | Summer 2025

Homework 2. Finish By 5/28.

Reading: Notes: Chapter 1, Chapter 2. **Videos:** 1.5, 1.6, 2.1, 2.2

Quick Calculations: Every homework will have a problem that focuses on quick calculations to help you get familiar with the mechanics of the concepts introduced that week. This will also help prepare you for exams, which will also include a similar problem.

Problem 2.1 (**Video 1.5**, **Lecture Problem**) Consider an experiment with sample space $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$. The outcomes have probabilities

$$\mathbb{P}[\{1\}] = \frac{1}{4} \quad \mathbb{P}[\{2\}] = \frac{1}{4} \quad \mathbb{P}[\{3\}] = \frac{1}{8} \quad \mathbb{P}[\{4\}] = \frac{1}{8}$$
$$\mathbb{P}[\{5\}] = \frac{1}{16} \quad \mathbb{P}[\{6\}] = \frac{1}{16} \quad \mathbb{P}[\{7\}] = \frac{1}{16} \quad \mathbb{P}[\{8\}] = \frac{1}{16} \ .$$

We also define the events

$$A = \{1, 3, 4\} \qquad B = \{2, 3, 4\} \qquad C = \{3, 4, 5, 6, 7, 8\}$$
$$D = \{2, 3, 5, 6\} \qquad E = \{2, 4, 6, 7\} \qquad F = \{5, 6, 7, 8\} \ .$$

For each of the following questions, give a "Yes" or "No" answer as well as your reasoning and calculations.

(a) Are the events A, B, and C independent? If not, are they at least pairwise independent?

(b) Are the events A and D independent?

(c) Are the events A and F independent?

(d) Are the events B and Ω independent?

(e) Are the events D, E, and F independent? If not, are they are least pairwise independent?

(f) Are the events A and D conditionally independent given C?

Problem 2.2 (**Video 1.5, 1.6**, **Lecture Problem**) Consider the following scenario. You play a simple game with probability of winning 1/4. You play this game repeatedly until your third loss, and then stop playing. Assume all games are independent.

(a) What is the probability of the following specific sequence of game outcomes: Win, Lose, Win, Lose, Lose?

(b) How many different sequences of games are there that end after exactly 5 games? (Hint: you must lose the last game to stop. There aren't that many, so you can enumerate them.)

(c) What is the probability of playing exactly 5 games?

(d) Given that you play exactly 5 games, what is the probability that your first game ended in a loss?

(e) Now, let's generalize this a bit. Say the probability of winning an individual game is p and that you play until your m^{th} loss. What is the probability of playing exactly k games?

Problem 2.3 (**Video 1.6**) You would like to evaluate the probability of success for testing a batch of n widgets. To start out, let's assume that if there is a problem with the batch, exactly 1 out of the n widgets are defective. You are willing to test only k of the widgets (due to budget or times constraints).

(a) How many ways are there of testing k out of n widgets?

(b) How many ways are there of testing k widgets with the defective widget included?

(c) Use your answers from parts (a) and (b) to determine the probability of catching a defective batch.

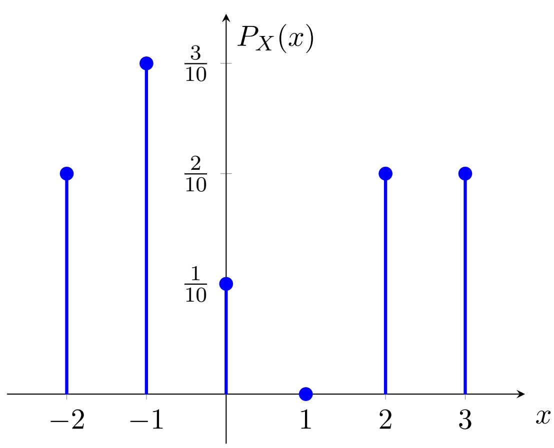
(d) Evaluate your answer from part (c) for $n = 20$ and $k = 5$.

(e) Now, say that a defective batch contains exactly 2 defective widgets. How many ways are there of testing k widgets with *at least one* defective widget included? (You may assume that $k > 2$.)

(f) Use your answer from part (e) to determine the probability of catching a defective batch.

(g) Evaluate your answer from part (f) for $n = 20$ and $k = 5$.

Problem 2.4 (**Video 2.1, 2.2**, **Lecture Problem**)



Consider the PMF above and let $A = \{-2, -1, 3\}$.

(a) Calculate the probability that X falls into A, $\mathbb{P}[X \in A]$.

(b) Calculate the probability that X^2 exceeds 1, $\mathbb{P}[X^2 > 1]$.

(c) Given that $\{X \in A\}$ occurs, what is the conditional probability that X^2 exceeds 1, $\mathbb{P}[X^2 > 1 | X \in A]$?

(d) Determine the CDF $F_X(x)$.

Problem 2.5 (**Video 1.5, 1.6, 2.1, 2.2**, **Quick Calculations**) Calculate each of the requested quantities.

(a) Let A and B be independent events with $\mathbb{P}[A] = 1/5$ and $\mathbb{P}[B] = 1/4$. Calculate $\mathbb{P}[A \cap B]$ and $\mathbb{P}[A \cup B]$.

(b) Let A_1, A_2, A_3 be events that are conditionally independent given B. *Additionally, assume that A_1, A_2, A_3 are conditionally independent given B^{c}.* Assume that $\mathbb{P}[A_i|B] = 1/4$ and $\mathbb{P}[A_i|B^{\mathsf{c}}] = 1/2$ for $i = 1, 2, 3$ and $\mathbb{P}[B] = 1/3$. Calculate $\mathbb{P}[A_1 \cap A_2^{\mathsf{c}} \cap A_3 | B]$ and $\mathbb{P}[A_1 \cap A_2^{\mathsf{c}} \cap A_3]$.

(c) Consider a packet of jellybeans that contains 9 jellybeans, of which 4 are lemon and the remaining 5 are raspberry. You reach in and pull out 3 jellybeans. What is the probability that they are all lemon? What is the probability that they are all raspberry?

(d) Let X be a random variable with PMF $P_X(x) = \begin{cases} 1/6 & x = -1, +1 \\ 2/3 & x = 0 \end{cases}$. Calculate $\mathbb{P}[X \neq 0]$ and $\mathbb{P}[X > 0 | X \neq 0]$.

(e) If the random variable Y has CDF $F_Y(y) = \begin{cases} 0 & y < 1 \\ 1/4 & 1 \leq y < 5 \\ 1 & 5 \leq y \end{cases}$, what is the PMF of Y?